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Quantum entanglement helps in improving economic efficiency

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Abstract
We propose an economic regulation approach based on quantum game theory for the government to reduce the abuses of oligopolistic competition. Theoretical analysis shows that this approach can help government improve the economic efficiency of the oligopolistic market, and help prevent monopoly due to incorrect information. These advantages are completely attributed to the quantum entanglement, a unique quantum mechanical character.

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1. Introduction

Information science has experienced a fundamental innovation since the last decades of the twentieth century through the combination of quantum physics, resulting in a new active field of research, quantum information science. Since information is of utmost importance in economy, it is of both fundamental and practical interest to investigate the economic behaviour from the perspective of quantum information. Quantum game theory [1–17] provides a useful and important basis to carry out such investigations because many economic phenomena could be, in nature, regarded as games [18].

A familiar phenomenon in economy is oligopoly, which is a market dominated by a few firms which are powerful enough to influence the market price. Oligopoly has low economic efficiency because it is a typical case of imperfect competition that leads to insufficient products, high selling price and finally reduced consumer satisfaction [19]. Things become worse when such inefficiency reaches a maximum, monopoly. Some well known models in economics to investigate the behaviour of oligopoly have been quantized and analysed from the perspective of quantum game theory, e.g. the quantum Cournot's Duopoly game

[10, 12], the quantum Bertrand Duopoly game [15] and the quantum Stackelberg Duopoly game [7]. While previous papers focused on the players' profits, in this paper we analyse Cournot's Duopoly game [20] emphasizing the favour of governments and propose a possible application of quantum game theory. Specifically, we propose an economic regulation approach based on quantum entanglement that can be used by governments to reduce abuses of oligopoly, which will help improve economic efficiency in situations of imperfect competition.

Here the original Cournot's Duopoly game with incorrect information and the model including the government's regulation based on quantum entanglement are referred to as the *classical model* and *quantum model* respectively.

2. The classical scenario

We now briefly recall the classical model and show the inefficient behaviour of the market. In Cournot's Duopoly game there are two oligopolistic firms (firm 1 and firm 2) producing homogeneous products. They simultaneously and respectively choose their quantities (strategies) q_1 and q_2. Then the market price is determined by $P(Q) = a - Q$ for $Q \leqslant a$ while $P(Q) = 0$ for $Q > a$, where $Q = q_1 + q_2$ is the total quantity and a is a constant. The unit costs of both firms are virtually the same, denoted by c. So the profit of each firm is $u_j(q_1, q_2) = q_j[P(Q) - c]$, with $j = 1, 2$. To simulate a possible case in a realistic market, here we introduce informational asymmetry into two firms, i.e. firm 1 thinks firm 2's unit cost as c', while firm 2 knows that firm 1's unit cost is c and that firm 1 has incorrect information about c'. Such incorrect information may lead to the strategy abuse of firm 1, which will cause instability of the market due to the influenced power of both firms. Note the model here is different from the one adopted in [12], in which firm 1 has correct but incomplete information about firm 2.

Let q_1^* and q_2^* be the virtual quantities of firm 1 and firm 2, respectively. Equipped with the incorrect information firm 1 takes its opponent as firm $2'$ (imaginary and non-existing). Let $q_{2'}^*$ be the quantity that firm 1 thinks firm 2 chooses. Firm 1 then will set $q_1 = q_1^*$ to maximize its (imaginary) profit, $u_1(q_1, q_{2'}^*) = q_1(a - q_1 - q_{2'}^* - c)$. In its imagination, firm 1 assumes that firm 2 would set $q_{2'} = q_{2'}^*$ to maximize the profit $u_{2'}(q_1^*, q_{2'}) = q_{2'}(a - q_1^* - q_{2'} - c')$. Solving these two optimization problems yields $q_1^* = \frac{1}{3}(1 - \sigma)k$, where $k = a - c$ and

$$\sigma = \frac{c - c'}{k}, \tag{1}$$

the absolute value of which can be regarded as the *informational incorrectness* of firm 1. Knowing the imagination of firm 1 and its chosen quantity (q_1^*), firm 2 will set its virtual quantity $q_2 = q_2^*$ to maximize its profit $u_2(q_2) = q_2(a - q_1^* - q_2 - c)$. Hence the virtual quantity of firm 2 is $q_2^* = \frac{1}{6}(2+\sigma)k$. Now the profits of the two firms are $u_1^* = \frac{1}{18}(2-\sigma-\sigma^2)k^2$ and $u_2^* = \frac{1}{36}(4 + 4\sigma + \sigma^2)k^2$.

But these mathematical results are not the actual results in the realistic market, for some realistic requirements have not been considered. The first requirement is that q_1^* and q_2^* must be positive, as they are quantities of the products. When either q_1^* or q_2^* happens to be negative or zero, it means the firm with negative or zero quantity has to exit the competition. Thus the remaining firm has a monopoly of the market. The second requirement is that if firm 2 were to have a higher profit in an informational symmetric situation, it will actively communicate with firm 1 to ensure $\sigma = 0$, because firm 2 has complete knowledge of the information. Following these requirements, it immediately finds out that only the situations $\sigma \geqslant 0$ need to be considered and the actual results of the competition are when $\sigma \in [0, 1)$, the quantities of

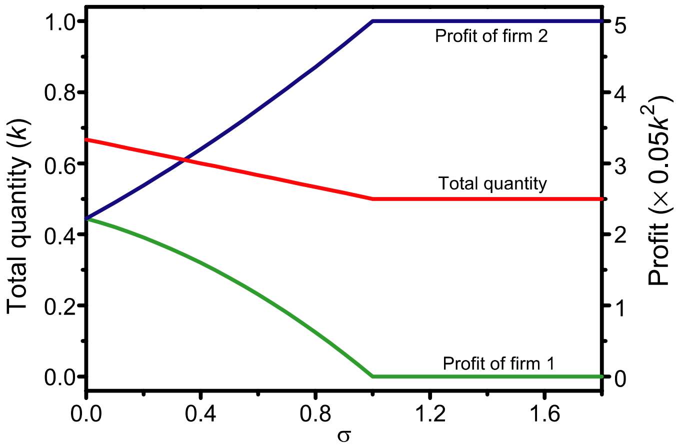

Figure 1. The total quantity and the profits of the two firms as functions of σ. The left and the right ordinates respectively represent the quantity and the profit scales.

the two firms are

$$q_1^* = \frac{1}{3}(1-\sigma)k, \qquad q_2^* = \frac{1}{6}(2+\sigma)k, \tag{2}$$

and the profits are

$$u_1^* = \frac{1}{18}(2-\sigma-\sigma^2)k^2, \qquad u_2^* = \frac{1}{36}(4+4\sigma+\sigma^2)k^2, \tag{3}$$

and when $\sigma \geqslant 1$, firm 1 exits from the market and

$$q_2^* = k/2, \qquad u_2^* = k^2/4, \tag{4}$$

where $k/2$ and $k^2/4$ are monopoly quantity and monopoly profit respectively.

So a monopoly appears when σ reaches 1. Meanwhile, as an indication of economic efficiency, the total quantity $Q = \frac{1}{6}(4-\sigma)k$ monotonical decreases with the increase of σ, until it reaches the minimum $(k/2)$ when a monopoly appears. In general, the economic efficiency becomes worse when σ increases, and a monopoly which leads to the worst economic efficiency occurs when $\sigma \geqslant 1$ (figure 1).

3. The quantum scenario

Government needs to prevent monopoly and improve economic efficiency. The proposed approach that can help government achieve these aims is based on a quantity-determining set (figure 2), which can be realized in feasible optical experiments and is the same as presented in [10]. The set involves the government's control and the strategic moves of the two firms, and it finally gives the quantities that the two firms should produce.

In figure 2, $|vac\rangle_1$ and $|vac\rangle_2$ represent the vacuum states of two single-mode electromagnetic fields. $\hat{S}(\gamma) = \exp\{-\mathrm{i}\gamma(\hat{a}^{+2}+\hat{a}^2)/2\}$ is the squeezing operator, where $\gamma \in (-\infty, +\infty)$ and $\hat{a}^+$ $(\hat{a})$ is the creation (annihilation) operator performed on the corresponding electromagnetic field. The squeezing operator can be implemented by parametric down-conversion inside a non-linear crystal [21]. All squeezing operators in this set share the same value of γ. BS is a beam splitter, whose operator is $\hat{B}(\theta,\phi) = \exp\left\{\theta\left(\hat{a}_1^+\hat{a}_2\,\mathrm{e}^{\mathrm{i}\phi} - \hat{a}_1\hat{a}_2^+\,\mathrm{e}^{-\mathrm{i}\phi}\right)/2\right\}$. Here we choose $BS_1 = BS_2 = \hat{B}(\pi/2, 3\pi/2)$ and $BS_3 = \hat{B}(\pi/2, \pi/2)$. $\hat{D}_j(x_j) = \exp(-\mathrm{i}x_j\hat{P}_j)$ ($j = 1, 2$ represent the two firms) can be realized by simple phase-space displacement on the corresponding electromagnetic field, where $x_j \in (-\infty, +\infty)$ and $\hat{P}_j$ is the 'momentum' operator. M denotes the final measurement.

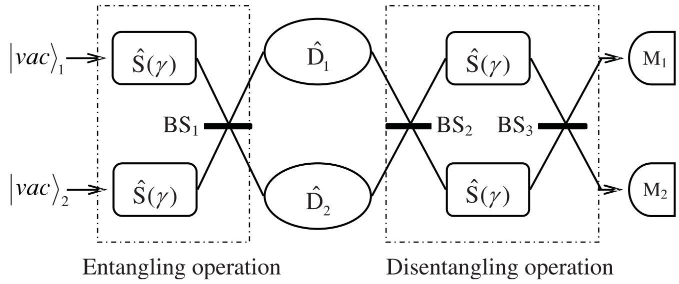

Figure 2. The quantity-determining set based on feasible optical experiments.

A typical run of the quantity-determining set is as follows. The inputs of the set are the two single-mode electromagnetic fields both in their vacuum states, which belong to the two firms. After the operations in the left dashed frame (see figure 2), the two electromagnetic fields become entangled. The degree of entanglement can be represented by γ, the parameter of the squeezing operator $\hat{S}(\gamma)$. Its value ($\gamma \in (-\infty, +\infty)$) is under the control of the government and is known to both firms. Then the two electromagnetic fields are sent to firms 1 and 2 respectively. The strategic move of each firm is associated with the local unitary operator $\hat{D}_j(x_j)$ performed on its individual electromagnetic field. The parameter x_j is freely and independently selected by each firm from the set $\{x_j \mid x_j \in (-\infty, +\infty)\}$. It is the strategy of each firm in this model instead of the quantity. After the disentangling operation which is realized by the operations in the right dashed frame, the two electromagnetic fields are forwarded to the final measurement. Measurement is carried out corresponding to the observable $\hat{X}_j$ ($j = 1, 2$) (the 'position' operators) for each electromagnetic field. The result of the measurement performed on the jth electromagnetic field gives the quantity that firm j should produce. Here we use the method in [10] to reduce the uncertainty in measurement, and presume the limit case that the uncertainty of $\hat{X}_j$ tends to zero.

There are some additional rules to abide by in implementing the quantity-determining set. First, government should inform both firms of the value of γ beforehand. Second, if the final measurement result shows that the quantity of one firm is negative or zero, government allows this firm to exit from the market and allows the other firm to redecide its quantity (not x_j). Under this situation the remaining firm will naturally choose the monopoly quantity to obtain the monopoly profit, i.e. a monopoly appears.

The general form of the measurement result in the quantity-determining set is $q_{1Q} = x_1 \cosh\gamma + x_2 \sinh\gamma$ and $q_{2Q} = x_2 \cosh\gamma + x_1 \sinh\gamma$, where x_1 and x_2 represent the strategies used by the two firms and the subscript 'Q' denotes 'quantum'. It is worth noting that the classical model can be recovered by choosing γ to be zero, since the two firms can directly decide their quantities now. The strategic space, i.e. $\{x_j \mid x_j \in (-\infty, +\infty)\}$, is the same for both $\gamma = 0$ (the classical situation) and $\gamma \neq 0$. So any novel features exhibited in the quantum model with non-zero entanglement are completely attributed to the quantum entanglement.

In the quantum model both firms should carefully choose their strategy x_j instead of choosing the quantities. Specifically, firm 1 will set $x_1 = x_1^*$ to maximize its profit

$$u_{1Q}(x_1, x_{2'}^*) = q_{1Q}(x_1, x_{2'}^*)[P(x_1, x_{2'}^*) - c], \tag{5}$$

thinking that firm 2 would choose $x_{2'}^*$ to maximize its profit

$$u_{2'Q}(x_1^*, x_{2'}) = q_{2'Q}(x_1^*, x_{2'})[P(x_1^*, x_{2'}) - c']. \tag{6}$$

Solving these two optimization problems yields

$$x_1^* = \frac{(1 - e^{2\gamma}\sigma)\cosh\gamma}{1 + 2e^{2\gamma}} k. \tag{7}$$

Firm 2 will choose $x_2 = x_2^*$ to maximize its profit

$$u_{2Q}(x_2) = q_{2Q}(x_2)[a - q_{1Q}(x_2) - q_{2Q}(x_2) - c], \tag{8}$$

where x_1^* in $q_{1Q}(x_2) = x_1^* \cosh\gamma + x_2 \sinh\gamma$ and $q_{2Q}(x_2) = x_2 \cosh\gamma + x_1^* \sinh\gamma$ is given in equation (7). Hence it yields

$$x_2^* = \frac{1 + e^{2\gamma} + e^{4\gamma}\sigma}{2e^{\gamma}(1 + 2e^{2\gamma})} k. \tag{9}$$

So the mathematical results of the quantities for the two firms in the quantum model are $q_{1Q}^* = x_1^* \cosh\gamma + x_2^* \sinh\gamma$ and $q_{2Q}^* = x_2^* \cosh\gamma + x_1^* \sinh\gamma$.

Taking the mentioned two realistic requirements into consideration, we also only need to discuss the situations of $\sigma \geqslant 0$ in the quantum model. When $\sigma \in \left[0, \frac{2(1+e^{2\gamma})}{1+3e^{2\gamma}}\right)$, the actual quantities of the two firms are

$$\begin{aligned} q_{1Q}^* &= \frac{1 + e^{2\gamma}}{4(1 + 2e^{2\gamma})} \left(2 - \frac{1 + 3e^{2\gamma}}{1 + e^{2\gamma}}\sigma\right) k, \\ q_{2Q}^* &= \frac{1 + e^{2\gamma}}{4(1 + 2e^{2\gamma})} (2 + \sigma) k, \end{aligned} \tag{10}$$

and the profits are

$$\begin{aligned} u_{1Q}^* &= \frac{e^{2\gamma}(1 + e^{2\gamma})}{8(1 + 2e^{2\gamma})^2} \left(4 - \frac{4e^{2\gamma}}{1 + e^{2\gamma}}\sigma - \frac{1 + 3e^{2\gamma}}{1 + e^{2\gamma}}\sigma^2\right) k^2, \\ u_{2Q}^* &= \frac{e^{2\gamma}(1 + e^{2\gamma})}{8(1 + 2e^{2\gamma})^2} (4 + 4\sigma + \sigma^2) k^2, \end{aligned} \tag{11}$$

when $\sigma \geqslant \frac{2(1+e^{2\gamma})}{1+3e^{2\gamma}}$, firm 1 exits the market and firm 2 has the monopoly. The quantity and profit of firm 2 are $k/2$ and $k^2/4$ respectively.

Let

$$\sigma_s(\gamma) = \frac{2(1 + e^{2\gamma})}{1 + 3e^{2\gamma}}, \tag{12}$$

thus $\sigma_s(\gamma)$ is the upper bound of σ, below which a monopoly will not appear. It is a decreasing function of γ and its maximum value 2 will be reached when $\gamma \to -\infty$. So as long as σ is below 2, the government can always prevent the occurrence of a monopoly by choosing a proper value of γ. Compared to the case of the classical model, the upper bound of σ is considerably extended from 1 to 2 by using our regulation approach.

The range of γ from which government can choose to prevent monopoly under σ is $(-\infty, +\infty)$ for $0 \leqslant \sigma < 2/3$ and $\left(-\infty, \frac{1}{2}\ln\frac{2-\sigma}{3\sigma-2}\right)$ for $2/3 \leqslant \sigma < 2$, which means the reduction of government's choices. The total quantity in such a range, i.e. when monopoly does not appear, is

$$Q = \frac{1 + e^{2\gamma}}{2(1 + 2e^{2\gamma})} \left(2 - \frac{e^{2\gamma}}{1 + e^{2\gamma}}\sigma\right) k. \tag{13}$$

It increases monotonically as γ decreases. When $\gamma \to -\infty$ it reaches its maximum value of k, which is the maximum total quantity we can expect from the market, since the profits of both firms have reduced to zero due to the extremely large total quantity. So long as $0 \leqslant \sigma < 2$,

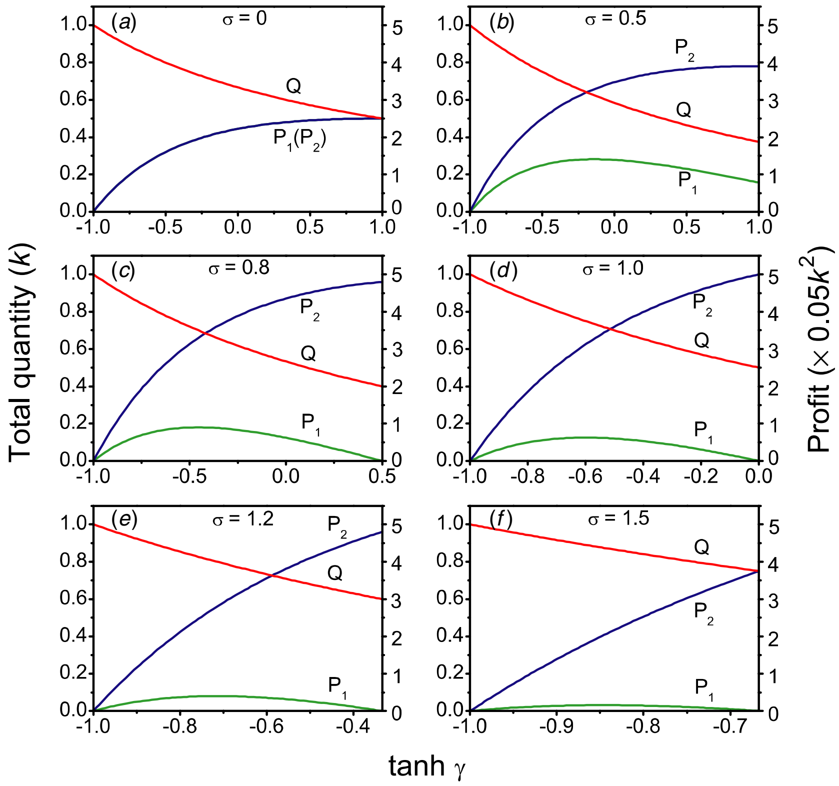

Figure 3. The total quantity (Q) and the profits of the two firms (P_1 for firm 1 and P_2 for firm 2) as functions of $\tanh\gamma$ for some selected values of σ. The left and the right ordinates of each figure respectively represent the quantity and the profit scales. The range of abscissa represents the range of γ that government can choose from to prevent monopoly under the corresponding σ.

the total quantity can always be increased to a sufficient level by choosing a proper value of γ (figure 3). Thus the economic efficiency is improved.

In figure 3 the profits of the two firms are also plotted. It shows that when the new regulation approach is implemented, the relative status of the two firms remains unchanged, i.e. firm 1 always profits less than firm 2 due to its incorrect information. They gain the same profits only when $\sigma = 0$ (figure 3(*a*)), for then the two firms are completely symmetrical.

4. Conclusion and discussion

We propose an economic regulation approach for government to reduce abuses of oligopolistic competition, which is built on quantum game theory. Theoretical analysis shows that this approach is effective in preventing monopoly and improving economic efficiency. Specifically, our approach considerably extends the region of σ (measure of informational incorrectness of firm 1) in which the monopoly will not appear from [0, 1) to [0, 2), and the total quantity of the products can always reach a sufficient level by choosing a proper value of γ (the degree of quantum entanglement) when a monopoly does not appear. Detailed analysis shows that such advantages are completely attributed to the quantum entanglement, a unique quantum mechanical character that has no classical correspondence.

Compared to the traditional regulation approaches of the government, our approach has two advantages. The first one is that, as the classical model is a subset of the quantum model, our approach provides a much more flexible way for government regulation. Government can choose whether or not to regulate the market simply by setting γ to be non-zero or zero. The second advantage is our approach offers an alternative to government to control the total quantity of products (equation (13)), which is sometimes classically achieved by government setting a limit. Further, our regulation results are stable, since this approach is still an outcome of competition and any individual deviation of the results (the equilibrium) only leads to the reduction of profit.

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